$\frac{84}{100}$

Birzeit University

Department of Mathematics

Second Hour Exam **Math 132** **Summer 2015**

Student name: [illegible] **Section..** 10:00 **Student no.:** [illegible]

Q#1 ~~60%~~ 72 circle the correct answer.

1. $\sum_{2}^{\infty} \frac{1}{n(n-1)}$ equals

(a) 4/3
(b) 1
(d) 3/4
(d) Diverges

$\frac{-1}{n} + \frac{1}{n+1}$ $\quad \frac{n+1-n}{n(n-1)}$ $\quad \frac{1}{n^2-n}$ $\quad = \frac{-n}{n} - \frac{n}{n-1}$

$\frac{1}{2.1} + \frac{1}{2.3} + \frac{1}{}$ $\quad \frac{n^2-n+n-1-n^2}{n^2}$

$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+ \cdots =$

$\frac{1}{n-1} - \frac{1}{n} = \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)$

$\frac{n-(n-1)}{(n-1)(n)}$

2) Which of the following series converges conditionally?

(a) 3 - 1 + 1/9 - 1/27 + ...

(b) $\frac{1}{1\times2} - \frac{1}{2\times3} + \frac{1}{3\times4} - \frac{1}{4\times5} + \dots$

(c) $1/2^2 - 1/3^2 + 1/4^2 - \dots$

56 (d) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$

$\sum_{n=0}^{\infty} 3\left(\frac{-1}{3}\right)^n$ $\quad = 3 - 1$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ $\quad \sum_{n=2}^{\infty} \frac{(-1)^n}{n^2}$ $\quad \sum \frac{(-1)^n}{n}$

3) If $\{s_n\} = \{1 + \frac{(-1)^n}{n}\}$, then

(a) $\{s_n\}$ diverges
(b) $\{s_n\}$ converges to zero
(c) $\{s_n\}$ converges to e^{-1}
(d) $\{s_n\}$ converges to 1

$(1-1) + \left(1+\frac{1}{2}\right) + \left(1-\frac{1}{3}\right)$

$\sum$ converge + $\sum$ diverge = diverge

4. The sequence $(a_n) = (1-\frac{1}{n^2})^n$

a) Converges to e^{-1}
b) Converges to e
c) Converges to 1
d) diverges

$= \left(1-\left(\frac{1}{n}\right)^2\right)^n$

$\left(1-\frac{1}{n}\right)^n\left(1+\frac{1}{n}\right)^n$

$e^{-1} \cdot e^{1} = 1$ $\quad -1 < x + \frac{1}{2} < 1$

$|r| < 1$

$|x+\frac{1}{2}| < 1$

5) The Series $\left(1-\frac{1}{2}+\frac{1}{4}\right)\frac{1}{6}+\cdots$ $\quad \sum_{n=0}^{\infty}\left(\frac{(-1)^n}{2n}\right)$

a) Convrges to L Where $0.46 \le L \le 0.66$
b) Convrges to L Where $0.50 \le L \le 0.75$
c) Convrges to L Where $1 \le L \le 1.5$
d) Diverges

$0.75 - \frac{1}{6}$ $\quad \frac{3}{4} - \frac{1}{6} = \frac{18-1}{24} = \frac{17}{24}$

6)Which of the following series converges?

(a) $\Sigma \frac{1}{n}$ $\Rightarrow$ diverge $\Rightarrow$ P-test

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ $\Rightarrow$ diverge $\Rightarrow$ P-test

(c) $\sum_{n=1}^{\infty} \frac{1}{10n^2+1}$ $\Rightarrow$ converge by D.C.T

(d) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ $\Rightarrow$ diverge by D.C.T

7) The series $\sum_{n=1}^{\infty} \frac{1}{e^n+\sqrt{n}}$

$\left(\frac{1}{e}\right)^n$ converge $\frac{1}{e^n+\sqrt{n}} < \left(\frac{1}{e}\right)^n$ by D.C.T with $\frac{1}{e^n}$

a) Converges by limit comparison test compared with $\sum_{n=1}^{\infty} \frac{1}{2n}$.

b) diverges by direct comparison test compared with $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n}}$.

c) Converges by direct comparison test compared with $\sum_{n=1}^{\infty} \frac{1}{e^n}$.

d) diverges by nth term test

8) $\sum_{1}^{\infty} (\ln(x))^n$ Converges If

$|\ln x| < 1$

$-1 < \ln x < 1$

$\frac{1}{e} < x < e$

a) $-1 < x < 1$

b) $0 < x < e$

c) $0 < x < 1$

d) $e^{-1} < x < e$

9. The series $\sum_{n=1}^{\infty} n \tan \frac{1}{n}$

$\lim_{n\to\infty} n \tan \frac{1}{n}$

$= \lim_{n\to 0} \frac{\tan n}{n} = 1 \neq 0$

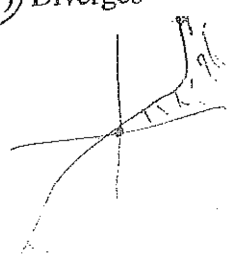
$\Rightarrow$ converge

a) Converges conditionaly

b) Converges absolutely

c) Converges by Integral Text

d) Diverges



$\lim_{a\to\infty} \int_1^a n \tan \frac{1}{n}$

y 1 0

tan ln cos

$\frac{\frac{2}{5}}{1-\frac{2}{5}} =$

$\frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{5}\cdot\frac{5}{4} = \frac{1}{4}$

$\frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12}$

10. The Series $\sum_{1}^{\infty}\frac{2^n-1}{5^n}$ $\qquad \left(\frac{2}{5}\right)^n - \left(\frac{1}{5}\right)^n$

a) Converges to $\frac{11}{12}$

b) Converges to $\frac{9}{12}$

c) Converges to 0

d) Converges to $\frac{5}{12}$

$\cancel{\left[\left(\frac{2}{5}\right)-\left(\frac{1}{5}\right)\right]+\left[\frac{4}{10}-\frac{1}{10}\right]+\left[\frac{8}{25}-\frac{1}{25}\right]}$

$\cancel{\frac{1}{5}+\frac{3}{10}+\frac{7}{25}}$

$\cancel{\frac{5}{10}+\frac{7}{25} = \frac{19}{25}}$

11) The radius of convergence of the series $\sum_{1}^{\infty}\frac{x^n}{2^n}$ is

a) R=1 $\qquad |x| < 1$

b) R=2

c) R=0 $\qquad -1 < x < 1$

d) R=∞

12) The series $\sum_{1}^{\infty}\frac{n^2}{e^n}$ $\qquad \frac{n^2}{e^n}$

a) Converges By nthterm Test $\qquad \lim \frac{(n+1)^2}{e^{n+1}}\cdot\frac{e^n}{n^2}$

b) Converges By Ratio Test

c) Diverges By Integral Test $\qquad = \frac{n^2+2n+1}{en^2} = \frac{1}{e} < 1$

d) Diverges by ratio test

converges

13) The sum of the series (2 - 1 + 1/2 - 1/4 + 1/8 - ...) is

(a) 4/3 $\qquad \sum_{n=0}^{\infty} 2\left(\frac{-1}{2}\right)^n$

(b) 5/4

(c) 3/2 $\qquad \left(\frac{2}{1-\frac{-1}{2}}\right) = \frac{2}{\frac{3}{2}} = \frac{4}{3}$

(d) ¾

14) The series $\sum_{1}^{\infty}\frac{2^n}{n^n}$ $\qquad \lim \sqrt[n]{\left(\frac{2}{n}\right)^n} = \infty > 1$

a) Diverges By nthroot Test $\qquad$ diverge by nth root test

b) Diverges By Direct Comparison Test With $\sum_{1}^{\infty}\frac{1}{n^n}$

c) Converges By Integral Test

d) Converges By nthroot Test

15. Consider $\sum_{1}^{\infty} a_n$ Where $a_n \geq 0$ Then

a) If $\lim_{x\to\infty} a_n = 0$ then $\sum_{1}^{\infty} a_n$ converges.

b) If $\sum_{1}^{\infty} a_n$ diverges then $\lim_{x\to\infty} a_n \neq 0$

c) If $\lim_{x\to\infty} a_n \neq 0$ then $\sum_{1}^{\infty} a_n$ diverges

d) If $\sum_{1}^{\infty} a_n$ converges then $\lim_{x\to\infty} a_n \neq 0$

16. Consider $I_1 = \int_{2}^{\infty} \frac{dx}{x^2}$ and $I_2 = \int_{0}^{\frac{1}{2}} \frac{dx}{\sqrt{x}}$ Then

converge

$\lim_{a\to 0^+} \int_a^{\frac{1}{2}} \frac{1}{\sqrt{x}}$

$= 2\sqrt{x} \Big]_a^{\frac{1}{2}}$

$= 2\sqrt{1/2} - 2\sqrt{a}$

a) Both Integrals Converge
b) Both Integrals Diverge
c) I_1 converges and I_2 diverges
d) I_2 converges and I_1 diverges

17. Which of the following sequences diverges?

(a) $\{\frac{(-1)^n}{n}\}$

(b) $\{\frac{5^n}{4^n + \sin n}\}$

(c) $\{n^2/e^n\}$

(d) $\{\sqrt[n]{10n}\}$

$\lim \frac{n^2}{e^n} = \lim \frac{2n}{e^n} = \lim \frac{2}{e^n} = \infty \neq 0$

diverge

18. $\sum_{1}^{\infty} (-1)^n \frac{\ln(n)}{n^3}$

$\frac{\ln n}{n^3} \leq \frac{n}{n^3}$

$\frac{\ln n}{n^3} < \frac{1}{n^2}$

$\frac{1}{n^2}$ converge p-test

$\Rightarrow \frac{\ln n}{n^3}$ converges

a) Is a geometric series
b) Converges conditionaly
c) Converges abslutely
d) Diverges

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1. (~~24~~%) Test for Convergence

a) $\int_1^\infty \frac{dx}{\sqrt{x^5+x}}$

$\frac{1}{\sqrt{x^5+x}} < \frac{1}{\sqrt{x^3+x}}$

since $\frac{1}{\sqrt{x^5+x}} < \frac{1}{\sqrt{x^4}} = \frac{1}{x^2}$

$\int_1^\infty \frac{1}{x^2} \cdot dx$ converge by p-test as $p = 2 > 1$

and

$\frac{1}{\sqrt{x^5+x}} < \frac{1}{\sqrt{x^3+x}}$

$\Rightarrow \int_1^\infty \frac{1}{\sqrt{x^5+x}} \cdot dx$ converge by D.C.T (Direct comparison test)

b) $\int_0^\infty \frac{\tan^{-1}x}{1+x^2}dx$

let $\tan^{-1} x = u$

$du = \frac{1}{1+x^2} \cdot dx$

$dx = 1+x^2 \cdot du$

$\int_0^\infty u \cdot du$

$= \lim_{a\to\infty} \int_0^a u \cdot du = \lim_{a\to\infty} \frac{u^2}{2} \Big]_0^a$

$= \lim_{a\to\infty} \frac{(\tan^{-1}x)^2}{2} \Big]_0^a = \lim_{a\to\infty} \frac{(\tan^{-1} a)^2}{2}$

$= \left(\frac{\pi}{2}\right)^2 = \frac{\pi^2}{4} \Rightarrow$ converge

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3. (~~16~~%) Consider the power series $\sum_{1}^{\infty} \frac{(-1)^n (x-2)^n}{n6^n}$ Find

a) Interval and radius of convergence

b)For what values of x does the series converges

1) absolutely 2) coditionally 3) diverge

$$\left|\frac{x-2}{6}\right| < 1$$

$$-1 < \frac{x-2}{6} < 1$$

$$-4 < x < 8$$

ⓐ The interval of convergence $[-4, 8]$

the radius of convergence = 6

-4 8

for $x = 1$

$$\sum_{1}^{\infty} \frac{(-1)^n (-1)^n}{n\,6^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{(-1)^n(-1)^n}{n\,6^n}} = \lim \frac{+1}{n^{\frac{1}{n}}\cdot 6} = \frac{+1}{6}$$

$$\left|\frac{+1}{6}\right| < 1$$

$\Rightarrow$ converge by n^{th} root test

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n(-1)^n}{n\,6^n}\right|} = \frac{-1}{6} \Rightarrow \left|\frac{-1}{6}\right| < 1 \Rightarrow$$ converge absolutely by n^{th} root test

for $x = 3$

$$\sum_{1}^{\infty} \frac{(-1)^n (1)^n}{n\,6^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{(-1)^n(1)^n}{n\,6^n}} = \lim_{n\to\infty} \frac{-1}{n^{\frac{1}{n}}\cdot 6} = \left|\frac{-1}{6}\right| < 1$$

$$\sum_{1}^{\infty} \frac{(-1)^n (1)^n}{n\,6^n} \Rightarrow \lim \sqrt[n]{\frac{(-1)^n(1)^n}{n\,6^n}} = \lim_{n\to\infty} \frac{1}{n^{\frac{1}{n}}\, 6} = \left|\frac{1}{6}\right| < 1$$

converge absolutely by n^{th} root test

ⓑ ① Converge absolutely $[-4, 8]$

② converge conditionally $\Rightarrow \{8\}$

③ diverge $(-\infty, -4) \cup (8, \infty)$

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